Intro to Calc

3.1 Exponential and Logistic Functions

Definitions

Exponential functions $f(x)=ab^x$

$a \neq 0$ and b is positive and $\neq 1$.
The constant a is the initial value (at $x=0$)
and b is the base.

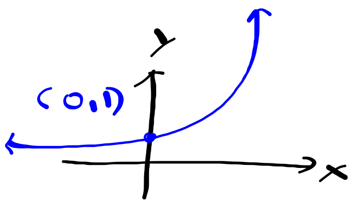
Exponential Growth and Decay

For any exponential function $f(x)=ab^x$

Exponential growth

a>0 and b>1, b is the growth factor.

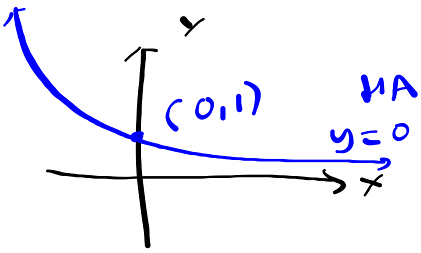
HA
$y=0$



$y = ab^{-x}$

Exponential decay

If a>0 and b<1, b is the decay factor.



Intro to Calc

Identifying Exponential functions

Determine whether or not the following are exponential functions. If so state the base and the initial value of the function.

a) $f(x) = -3(5)^x$

yes
$b = 5$
$a = IV = -3$

(b) $f(x) = 8x^9$

NO
variable not as exponent

(c) $f(x) = 15^x$

yes
$a = 1$
$b = 15$

(d) $f(x) = 7(2^{-x})$

yes
$a = 7$
$b = 2^{-1} = \frac{1}{2}$

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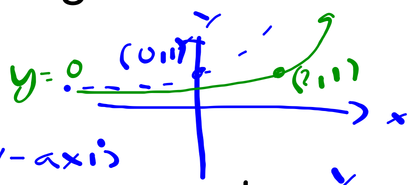
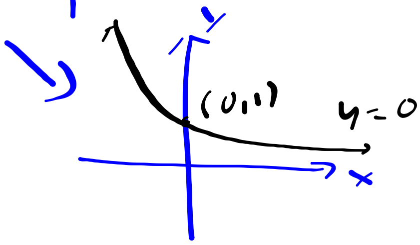
Transforming Exponential Functions

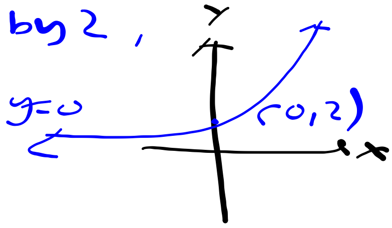
Describe how to transform the graph of $f(x)=3^x$ into the graph of the functions given below.

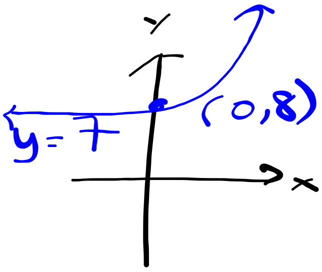
- $g(x)=3^{x-2}$ $\longrightarrow$ Right
- $h(x)=3^{-x}$ $\longrightarrow$ Reflect over y-axis
- $k(x)=2(3^x)$ $\rightarrow$ vertical stretch by 2,

$m(x)=3^x+7$









Using tables to find exponential functions

Ex. 2

Determine the exponential function represented by the data in the table.

x	f(x)
-2	3/4
-1	3/2
0	3
1	6
2	12

$y = ab^x$

$3 = ab^0$

$3 = a$

$y = 3b^x$

$6 = 3b^1$

$2 = b$

$y = 3(2^x)$

TRY

Determine the exponential function represented by the data in the table.

x	g(x)
-2	18
-1	6
0	2
1	2/3
2	2/9

$g(x) = 2b^x$

$6 = 2b^{-1}$

$3 = b^{-1}$

$3 = \frac{1}{b}$

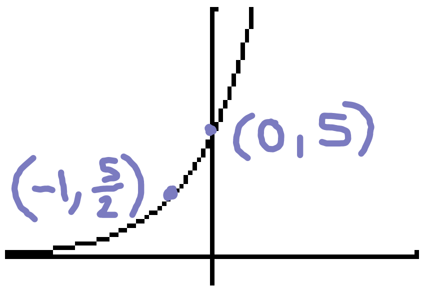
$b = \frac{1}{3}$

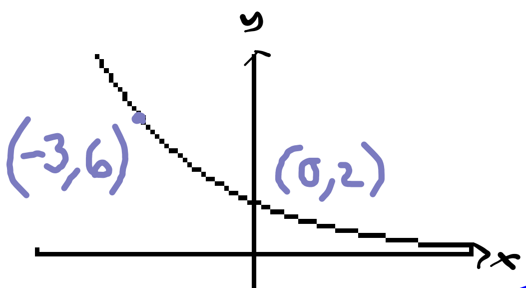
$y = 2(\frac{1}{3})^x$

$y = 2(3^{-1})^x$

$g(x) = 2(3^{-x})$

Given the graph below, determine a formula for the exponential function.





$y = 2b^x$

$6 = 2b^{-3}$

$3 = b^{-3}$

$3 = \frac{1}{b^3}$

$3b^3 = 1$

$b^3 = \frac{1}{3}$

$b = \frac{1}{\sqrt[3]{3}}$

$\sqrt[3]{\frac{1}{3}} = \frac{\sqrt[3]{1}}{\sqrt[3]{3}}$

$(3)^{-\frac{1}{3}} = (b^{-3})^{-\frac{1}{3}}$

$3^{-1/3} = b$

$\underline{y = 2(3^{-\frac{x}{3}})}$

$y = 2\left(\frac{1}{\sqrt[3]{3}}\right)^x$

$= 2(\sqrt[3]{3})^{-x}$

$2(3^{1/3})^{-x}$

$\underline{y = 2(3^{-\frac{x}{3}})}$

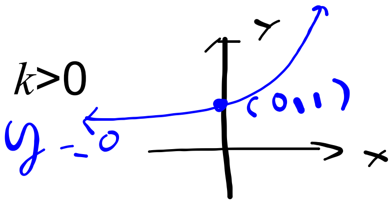
THE Exponential Function

$e \approx 2.718...$

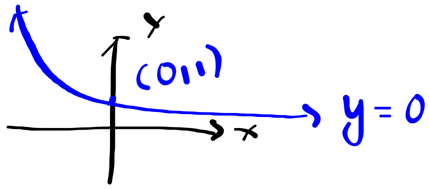
Any exponential $f(x)=ab^x$ can be rewritten as $f(x)=ae^{kx}$ for an appropriately chosen constant ($a>0$).

$b = e^k$

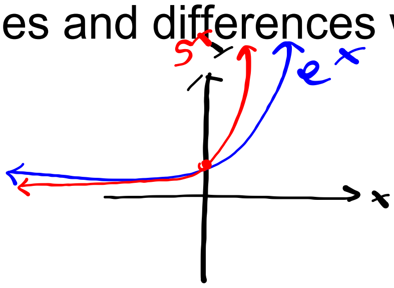
exponential growth $k>0$



exponential decay $k<0$



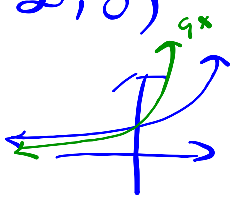
On your calculator graph $y=e^x$ and $y=5^x$. Discuss the similarities and differences with your neighbor.



5^{-1} e^{-1}

$\frac{1}{5}$ $\frac{1}{e} \approx \frac{1}{2.718}$

$e^x>5^x$ where? $(-\infty, 0)$



$7^x \leq 9^x$

$[0, \infty)$

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Transforming Exponential Functions

Describe how to transform the graph of $f(x)=e^x$ into the graph of the functions given below.

- $g(x)=e^{2x}$
- $h(x)= e^{-x}$
- $k(x)= 5e^{x+3}$

Graph the function $f(x)=4(e^x)+1$ and analyze it for domain, range, continuity, increasing and decreasing behavior, symmetry, extrema, asymptotes and end behavior.

$f(x) = 4e^x + 1$

Vertical stretch by 4

Up 1

D: $(-\infty, \infty)$

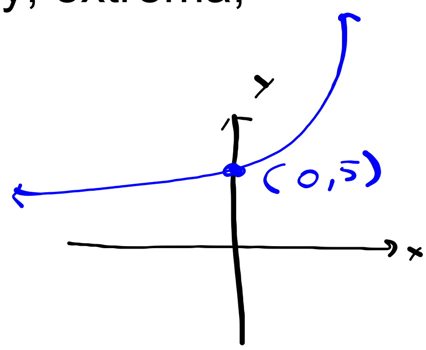
R: $(1, \infty)$

Inc $(-\infty, \infty)$

$\lim_{x \to \infty} f(x) \to \infty$

$\lim_{x \to -\infty} f(x) \to 1$

$y = 1$



Bounded below by 1.